BBC RD 1976/10

BBC

RESEARCH DEPARTMENT

1969 1974
THE QUEEN'S AWARD TO INDUSTRY

REPORT

DIGITAL VIDEO: a theoretical assessment of the quantising noise spectra arising from a change in the sampling frequency of PAL signals

T.A. Moore, B.E., M.Eng.Sc.

Research Department, Engineering Division
THE BRITISH BROADCASTING CORPORATION May 1976

BBC RD 1976/10
UDC 621.376.56:
621.397.132:
621.391.883.2

DIGITAL VIDEO: A THEORETICAL ASSESSMENT OF THE QUANTISING NOISE SPECTRA ARISING FROM A CHANGE IN THE SAMPLING FREQUENCY OF PAL SIGNALS

T.A. Moore, B.E., M.Eng.Sc.

Summary

When a digital video signal undergoes a change of sampling frequency the quantising noise of the original signal is modified by the re-sampling process, and extra quantising noise is added when the resultant samples are truncated to the required output accuracy.

In general the resulting quantising noise spectrum depends on the complete digital history of the signal and is determined by such factors as the original and final quantising accuracies, the interpolation profile and, in the case of sub-Nyquist sampling, the form of the comb-filter response.

In this report the resulting quantising noise spectra are derived for various combinations of sampling frequencies, super- and sub-Nyquist. The change in the signal-to-noise ratio is evaluated for PAL System I parameter values. Particular cases of interest are examined, for example when equal quantising accuracies are used for the input and output signals and when the input and output frequencies are combinations of four times, three times, and twice colour subcarrier frequency.

Issued under the authority of

P. Lange

Head of Research Department

Research Department, Engineering Division,
BRITISH BROADCASTING CORPORATION

May 1976

(EL-115)

DIGITAL VIDEO: A THEORETICAL ASSESSMENT OF THE QUANTISING NOISE SPECTRA ARISING FROM A CHANGE IN THE SAMPLING FREQUENCY OF PAL SIGNALS

Section	Title	Page
	Summary	Title Page
1.	Introduction	1
2.	Quantising noise due to a single sampling and quantising operation	1
3.	Super-Nyquist sampling	1
	3.1. Increase in sampling frequency	2
	3.2. Decrease in sampling frequency	2
	3.3. Typical values of increase in quantising noise	2
4.	Super- to sub-Nyquist sampling	3
	4.1. Four-times- to twice-subcarrier sampling	3
	4.2. Super-Nyquist to twice-subcarrier sampling	4
	4.3. Typical values of increase in quantising noise	4
5.	Sub- to super-Nyquist sampling	5
	5.1. Twice- to four-times-subcarrier sampling	5
	5.2. Twice-subcarrier to super-Nyquist	5
	5.3. Typical values of increase in quantising noise	5
6.	Conclusions and discussion of results	6
7.	References	6
8.	Appendix	7

DIGITAL VIDEO: A THEORETICAL ASSESSMENT OF THE QUANTISING NOISE SPECTRA ARISING FROM A CHANGE IN THE SAMPLING FREQUENCY OF PAL SIGNALS

T.A. Moore, B.E., M.Eng.Sc.

1. Introduction

In certain cases it may be required to change the sampling frequency of a digitally coded video signal. This may be necessary because of differences in the preferred choice of sampling frequencies. For example twice colour subcarrier frequency may be preferred in certain cases for transmission,[1] particularly in low-capacity channels, while in studios three times colour subcarrier may be preferred. Four times subcarrier may nevertheless be favoured as an alternative source-coding frequency since it permits relatively simple digital conversion to a twice-subcarrier signal.

Converting a given digital signal with one sampling frequency to a digital signal with a different sampling frequency without an intermediate analogue stage should degrade the signal less than decoding to analogue form and reading, although, in respect of quantising noise alone, there may be no advantage.

In this report a theoretical estimate is given of the spectrum of the quantising noise following a change in sampling frequency in a digital video system. The resulting spectra depend on the original quantising accuracy, i.e. the number of quantum levels, on the manner in which interpolation bandwidths are related to the sampling frequencies, and on the accuracy of the output 'numbers'; in some cases quantising noise due to 'aliasing' occurs, especially when the video signal is sampled at a sub-Nyquist rate.

Uniform quantising of a signal at regular time intervals is assumed throughout: this corresponds to straightforward pulse-code modulation (p.c.m.).

2. Quantising noise due to a single sampling and quantising operation

This case is dealt with first because it allows a 'before and after' comparison of the quantising-noise spectra and because the quantising noise following a chain of sampling operations is assumed to be the superposition of each of the contributions due to each stage.

Two operations are involved in converting an analogue signal to pulse-code-modulation form: quantisation and sampling (time quantisation). Conceptually, the order in which these take place is immaterial.

It is fairly well established that if quantisation alone is considered the total quantising-noise power is $\Delta^2/12$ where Δ is the quantisation step[3]; this follows from considering the noise value to be uniformly distributed between $\pm\Delta/2$. Because of the infinitely sharp 'jumps' of amount Δ in the quantising-error waveform, the quantising-noise spectrum extends far beyond the signal spectrum. The randomness and hence the 'whiteness' of the quantising error increases as the quantising accuracy is increased. This may be seen from Ref. 2, Fig. 4; for example, when 8-bit coding of a band-limited noise signal (of r.m.s. level 12 dB less than the peak value in the coding range), the spectral density does not fall to one-tenth of its low-frequency value until frequency values in excess of about 330 times the signal bandwidth are reached.

When the signal is sampled (at f_s) the sampled quantising-noise spectral density is different from that prior to sampling; because of the extent of the unsampled quantising noise spectrum, the sampled quantising noise is made up of a large set of aliasing components, which repeat periodically with frequency interval f_s. The sampled quantising-noise power of interest is that portion lying in the Nyquist frequency band 0 to $f_s/2$. In Appendix 8.1 the rather surprising result* is derived that the total sampled quantising-noise power in the Nyquist frequency band is equal to the total pre-sampled quantising noise power $\Delta^2/12$. If the mathematics look formidable, a glance at Fig. 7 should give the general idea.

Bearing in mind the uniform spectrum of quantising-noise before sampling, the spectrum of the quantising-noise after sampling will also be essentially flat. Since the noise power in the band 0 to $f_s/2$ after sampling is $\sigma^2 = \Delta^2/12$, the quantising-noise spectral density is then

$$2\sigma^2/f_s.$$

If the signal has a bandwidth of f_b, then the total noise-power in the signal frequency band is therefore

$$2\sigma^2 f_b/f_s.$$

A different approach is required when the video signal is slowly varying; 'contouring' effects then predominate on the picture and the quantising noise is not spectrally uniform. In this case dither is often used to break up contouring effects[2] and a flat noise-spectrum may be assumed as mentioned above.

3. Super-Nyquist sampling

At this stage only sampling frequencies which are greater than twice the highest signal baseband-frequency are considered. This avoids, for the present, any complications caused by the use of comb filtering to remove some of the

* Appendix 1 in Bennett's 1948 Paper[4] also derives this result. It points out what limitations must be imposed on the noise signal for the result to be valid. Briefly, the noise must not contain any discrete frequency components at multiples of $f_s/2$ or which differ by multiples of $f_s/2$.

aliased signal components in the baseband region. Two cases of conversion are considered: with an increase in f_s and with a decrease in f_s.

3.1. Increase in sampling frequency

The two sampling frequencies are denoted by f_1 (before) and f_2 (after) where $f_1 < f_2$.

If Δ_1 is the quantising step employed when sampling at f_1 then the power spectral density of the quantising-noise is

$$\frac{\sigma_1^2}{f_1} \text{ where } \sigma_1^2 = \Delta_1^2/12.$$

In the interpolation process in the frequency changer, this is band limited to $f_n = f_1/2$, and re-sampled at a rate f_2. If Δ_2 is the truncation (quantising) step of the output digital signal then the output noise spectra are:

$$\frac{\sigma_1^2}{f_1} \text{ for } |f| < \frac{f_1}{2}, \text{ plus}$$

$$\frac{\sigma_2^2}{f_2} \text{ for } |f| < \frac{f_2}{2} \text{ where } \sigma_2^2 = \frac{\Delta_2^2}{12}$$

Thus the total noise power in the baseband is

$$2\sigma_1^2 \frac{f_b}{f_1} + 2\sigma_2^2 \frac{f_b}{f_2},$$

since $f_b < \frac{1}{2}f_1 < \frac{1}{2}f_2$ for super-Nyquist sampling.

Thus, where equal quantising accuracies are used, the noise power is increased by a factor of

$$1 + \frac{f_1}{f_2},$$

by the re-sampling operation.

3.2. Decrease in sampling frequency

The same notation is used here, now with $f_1 > f_2$. To avoid undersampling, the interpolator frequency is chosen as the lower Nyquist frequency, $\frac{1}{2}f_2$: the total quantising noise power is

$$2\sigma_1^2 \frac{f_b}{f_1} + 2\sigma_2^2 \frac{f_b}{f_2} \qquad \text{as before.}$$

It may be instrumentally convenient to choose values of interpolator bandwidths (f_n) other than $\frac{1}{2}f_2$, e.g. f_b or $\frac{1}{2}f_1$. If $f_n > \frac{1}{2}f_2$, some aliasing occurs in the band below $\frac{1}{2}f_2$; however the noise power in the baseband is not affected if f_n lies in the range f_b to $f_2 - f_b$.

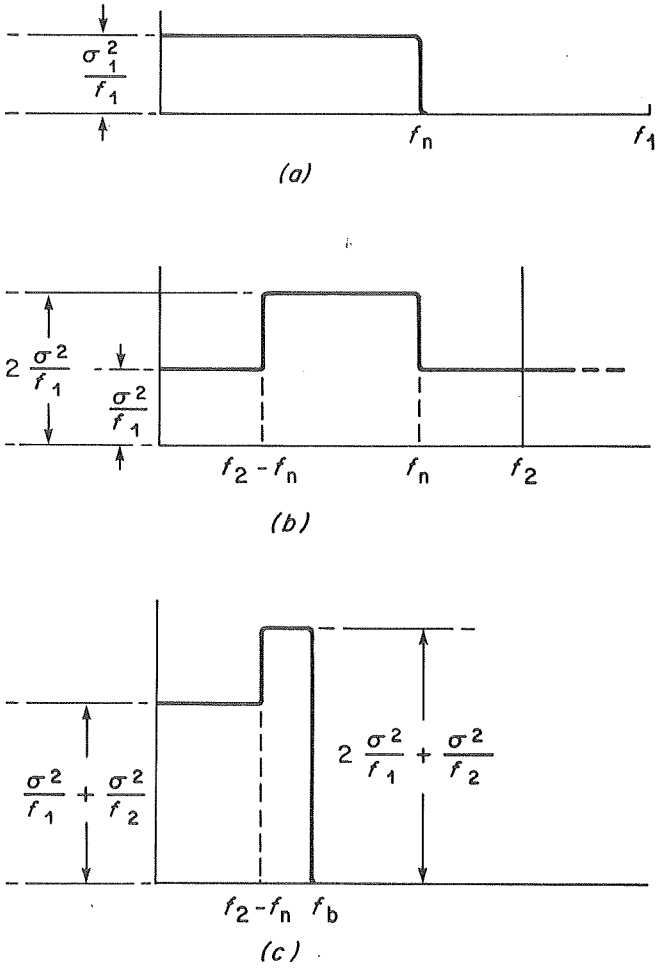


Fig. 1 - Decrease in super-Nyquist sampling frequency: noise spectra

(*a*) Quantising-noise spectrum, sampled at f_1, bandlimited to f_n.

(*b*) Signal in (*a*), sampled at f_2

(*c*) Spectrum of (*b*), plus truncation noise due to sampling at f_2, all bandlimited to f_b

If $f_n > f_2 - f_b$, as shown in Fig. 1(*a*) the baseband noise after sampling at f_2 is then as shown in Fig. 1(*b*)

$$2\sigma_2^2 \frac{f_b}{f_2} + 2\sigma_1^2 \frac{(f_2 - f_n)}{f_1} + 4\sigma_1^2 \frac{(f_b + f_n - f_2)}{f_1},$$

with a spectral density as shown in Fig. 1(*c*).

The noise power is again increased by a factor of

$$1 + \frac{f_1}{f_2}$$

due to resampling when equal quantising accuracies are used, provided that the interpolator cut-off frequency is chosen to avoid undersampling ($f_n < f_2 - f_b$).

3.3. Typical values of increase in quantising noise

In digital colour television systems, f_s is generally a multiple of colour subcarrier frequency f_{sc} (4·43 MHz for

PAL System I). The video bandwidth is 5·5 MHz. The two cases of interest in super-Nyquist sampling therefore are

(i) $f_1 = 3f_{sc}$, $f_2 = 4f_{sc}$

and (ii) $f_1 = 4f_{sc}$, $f_2 = 3f_{sc}$

Using these values, when the sampling frequency is increased from 3 f_{sc} to 4 f_{sc} the total quantising-noise power in the baseband is increased by 2·43 dB if $\Delta_1 = \Delta_2$ (i.e. a similar quantising accuracy is applied to both input and output signals); if the sampling frequency is reduced from 4 f_{sc} to 3 f_{sc} the total quantising-noise power in the baseband is increased by 3·68 dB, provided the interpolator bandwidth is between 5·5 MHz and 7·8 MHz, i.e. $(f_2 - f_b) > f_n > f_b$.

Under the same assumption a 3 f_{sc} sampled signal derived from a 4 f_{sc} sampled signal will have 2·43 dB more noise than a directly coded 3 f_{sc} signal; similarly a 4 f_{sc} signal derived from a 3 f_{sc} signal will have 3·68 dB more noise than a directly coded 4 f_{sc} signal.

4. Super- to sub-Nyquist sampling

When sub-Nyquist sampling of a video signal is used, it is important to be able to reject alias components by suitable comb filtering. The manner in which this is done depends critically on the relationship between line frequency and sampling frequency. When a composite colour-coded PAL signal is sampled at less than twice the highest video frequency the chosen sampling frequency is precisely twice colour-subcarrier frequency (2 f_{sc}). In general pre- and post-filtering is used in the system.

A convenient method of generating a 2 f_{sc} signal is firstly to generate or to convert to a signal sampled at four times subcarrier frequency (4 f_{sc}), comb filter this signal digitally* and take every second sample. For this reason in this section two cases are considered – a signal sampled at 4 f_{sc} and converted to 2 f_{sc} sampling, and the more general case of a signal sampled (super-Nyquist) at f_1 and converted to 2 f_{sc}, with 4 f_{sc} sampling used as an intermediate.

4.1. Four-times- to twice-subcarrier sampling

The quantising noise of the digital signal sampled at 4 f_{sc} has a power spectral density of ${\sigma_1}^2/4\,f_{sc}$ where ${\sigma_1}^2 = {\Delta_1}^2/12$. The comb filter characteristic is shown in Fig. 2: the nulls in the response in the vicinity of colour subcarrier are spaced apart by an amount equal to line frequency f_L. Colour subcarrier frequency $f_{sc} = (N - \frac{1}{4})f_L$, where $N = 284$, ignoring the 25 Hz effect.

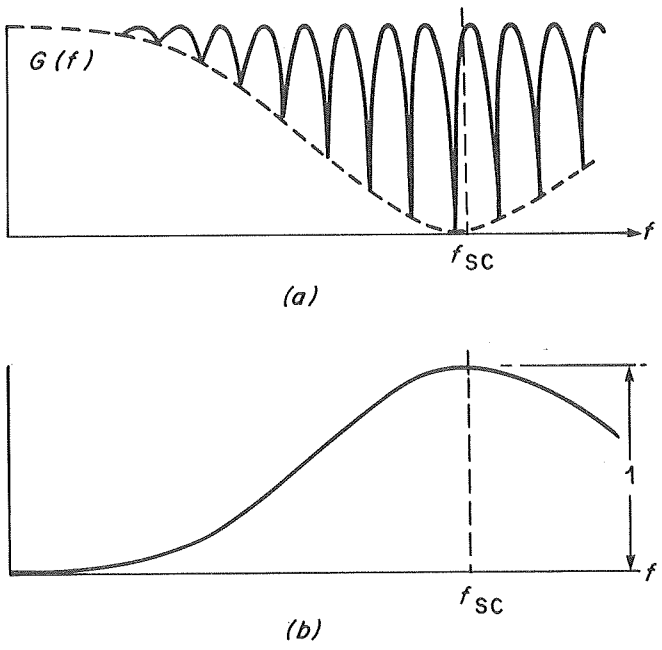


Fig. 2 - General form of comb-filter response
(*a*) Comb-filter frequency-response (*b*) Notch profile, $h(f)$

If $G(f)$ is the comb filter amplitude-frequency response then $G^2(f)$ is of the form

$$[1 - h(f) + \tfrac{1}{2}h^2(f)] + h(f)\,[1 - \tfrac{1}{2}h(f)]\,\cos 2\pi\frac{f}{f_L}$$

where $h(f)$ is a slowly varying function of frequency and $h(0) = 0$, $h(f_{sc}) = 1$. After passage through this comb filter the quantising noise spectral density is, ${\sigma_1}^2 G^2(f)/4f_{sc}$.

As mentioned above a 2 f_{sc}-sampled signal may be generated by simply extracting every second sample from the comb-filtered 4 f_{sc}-sampled signal; this obviates the need to interpolate and resample, as would be the case if the ratio of the two sampling frequencies were not an integer. However, this action is equivalent to band limiting the 4 f_{sc} signal to 4 $f_{sc}/2$ i.e. 2 f_{sc}, and then sampling at 2 f_{sc}.

The noise spectral density in the sub-sampled (2 f_{sc}) signal then has a spectral density*

$$\frac{{\sigma_1}^2}{4f_{sc}}\left\{G^2(f) + G^2(2f_{sc} - f)\right\}$$

Since $$\cos 2\pi\frac{(2f_{sc} - f)}{f_L} = -\cos 2\pi\frac{f}{f_L}$$

* It is assumed for the purposes of analysis that the comb filter does not itself truncate the output samples, i.e. it preserves the maximum accuracy possible given the limited quantising accuracy of the input signal.

* If a lower interpolation bandwidth (f_n) were used the alias components of the quantised noise centred around 2 f_{sc} would extend down as far as (2 $f_{sc} - f_n$), not d.c. as assumed above. This would result in a noise advantage, as discussed in Section 4.3.

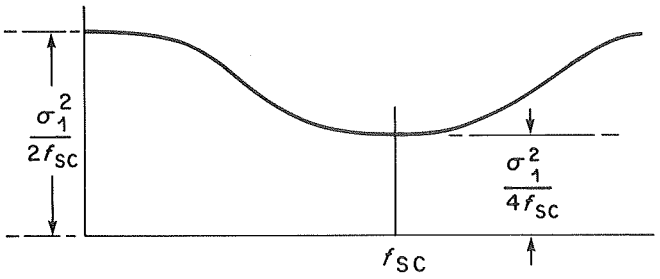


Fig. 3 - Spectrum of flat quantising noise, comb-filtered and sampled at 2 f_{sc}

because of the relationship between f_{sc} and f_L, this expression simplifies to

$$\frac{\sigma_1^2}{2f_{sc}}\left\{1 - h(f) + \tfrac{1}{2}h^2(f)\right\}$$

which is a smoothly varying curve, having a value of $\sigma_1^2/2f_{sc}$ at low frequency and a value of $\sigma_1^2/4f_{sc}$ around colour subcarrier frequency, as shown in Fig. 3.

At this stage truncation noise of spectral density $\sigma_2^2/(2f_{sc})$ is added to the noise spectrum. Both spectra are then modified by the output comb filter (which also has nulls at odd multiples of half line frequency) and the video low-pass output filter. The total noise in the base-band can then be evaluated by integrating the spectral densities with respect to frequency over the baseband: however, because of the differing visibility of noise at multiples of f_L and at multiples of $(f_L + \frac{1}{2}f_L)$, this figure is not likely to give a good estimate of the subjective 'annoy-ance' of the noise.

4.2. Super-Nyquist to twice-subcarrier sampling

This is a more general situation than that just analysed; however in the following paragraphs the original sampling frequency f_1 is taken to be *less* than $4f_{sc}$, which is used as the intermediate sampling frequency.

The quantising noise arising from sampling at f_1 has a power spectral density σ_1^2/f_1 in the region 0 to f_n (see Fig. 4(*a*)), after interpolation in the intermediate conversion to $4f_{sc}$ sampling. When this is comb filtered and sampled at $2f_{sc}$ the resultant spectrum is of the form shown in Fig. 4(*b*): this is in turn comb-filtered at the d.a.c. output, and the noise power is then equal to that portion of the noise in the video baseband.

In addition, the frequency down-conversion generates truncation noise of spectral density $\sigma_2^2/2f_{sc}$ which in turn is comb-filtered and video filtered.

4.3. Typical values of increase in quantising noise

To allow typical increases in quantising-noise power to be estimated a simple form of comb filter profile is assumed, in which negligible comb filtering occurs below a certain video frequency f_a (i.e. $h(f) = 0$) and above this

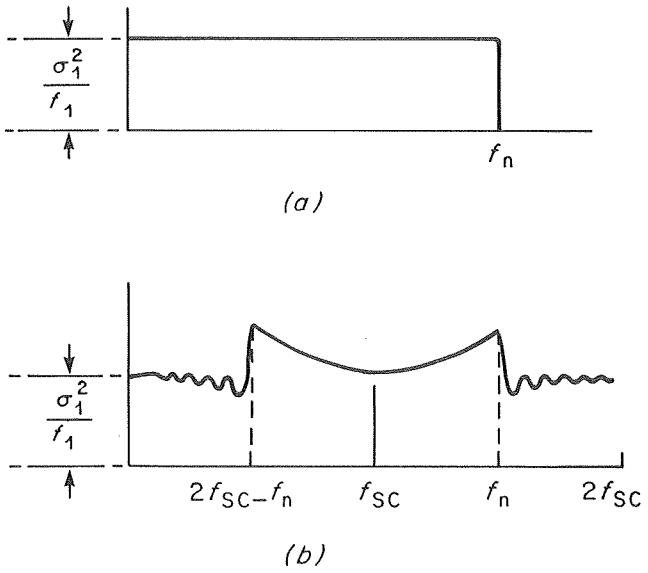


Fig. 4 - Form of noise spectrum in super- to sub-Nyquist ($2f_{sc}$) sampling
(*a*) Quantising noise from sampling at f_1, bandlimited to f_n
(*b*) Spectrum of (*a*), comb-filtered, and sampled at $2f_{sc}$

frequency the notches are full depth ($h(f) = 1$), as shown in Fig. 5(*a*). In addition the effect of the comb filter in the output $2f_{sc}$ filter is ignored since this comb filter is expected to have little effect on the noise visibility.

Equal quantising accuracies are assumed throughout,* and f_a is taken to be 2·5 MHz. Then a $2f_{sc}$-sampled signal derived from a $4f_{sc}$-sampled signal by taking alternate samples will have 5·38 dB more noise than the original $4f_{sc}$ signal and 2·37 dB more than a directly coded $2f_{sc}$ signal; Fig. 5(*b*) shows the quantising-noise spectrum after sampling at $2f_{sc}$ (but prior to truncation of the $2f_{sc}$ samples). Some noise advantage could be gained however by digitally band-limiting the $4f_{sc}$ signal using an interpolation profile of a lower bandwidth (than 8·86 MHz): at an interpolation bandwidth of 6·36 MHz ($= 2f_{sc} - f_a$) the quantising-noise spectrum after sampling at $2f_{sc}$ is flat and the total noise increase is 4·77 dB relative to the $4f_{sc}$ signal (and 1·76 dB relative to a directly coded $2f_{sc}$ signal). As may be seen from Fig. 5(*c*), in which a value of f_n between 6·36 MHz and 8·86 MHz is assumed, the noise decreases as f_n is decreased.

If the original signal is a $3f_{sc}$-sampled signal, the $2f_{sc}$-sampled signal has a total noise power in the video band of 3·98 dB more than the original if the interpolation bandwidth in the $3f_{sc}$ to $4f_{sc}$ intermediate conversion equals 6·36 MHz (with f_a = 2·5 MHz); the noise spectrum is then flat. If an interpolation bandwidth of 6·65 MHz ($= f_1/2$) is used, the noise increase is 4·16 dB; the resulting noise spectrum is shown in Fig. 6. Relative to a directly coded $2f_{sc}$ signal the noise increases are 2·22 dB and 2·40 dB respectively.

* When a $3f_{sc}$ sampled signal is converted to a $2f_{sc}$ signal through a $4f_{sc}$ signal, the quantising accuracy of the intermediate $4f_{sc}$ signal must generally be higher than that of the other signal; this aspect is discussed in Section 6.

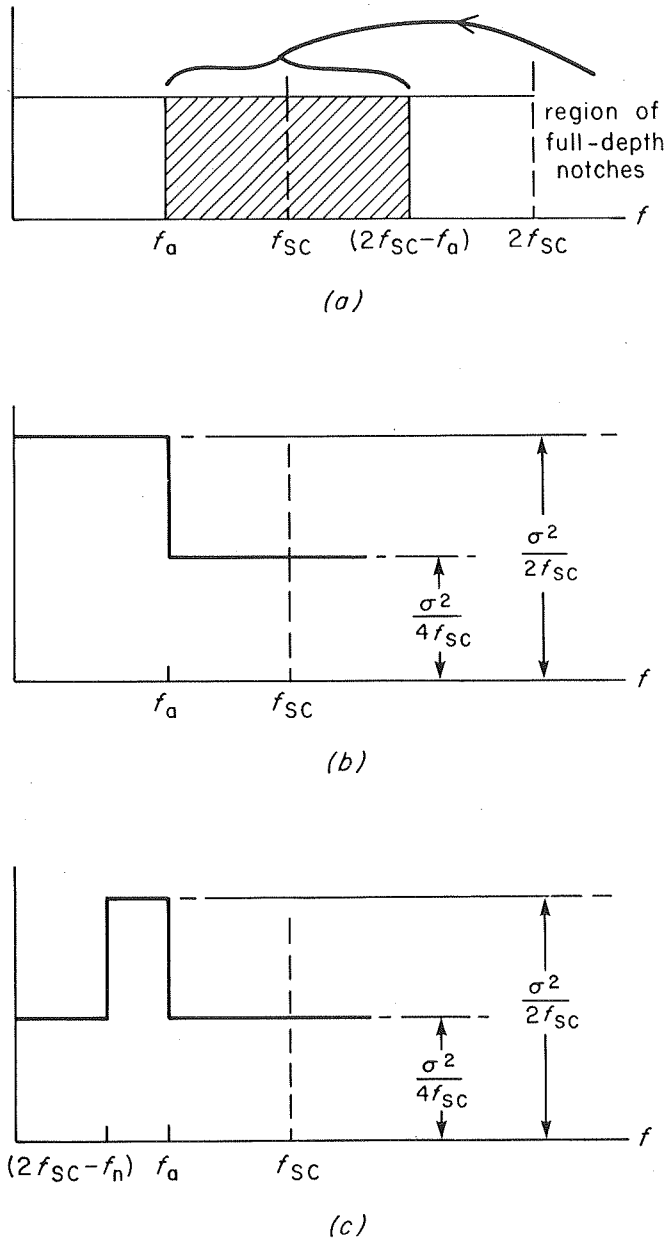


Fig. 5 - 4 f_{sc} to 2 f_{sc} conversion using 'simple' comb-filter
(*a*) Comb-filter frequency-response
(*b*) Resulting noise spectrum: no interpolation ($f_n = 2f_{sc}$)
(*c*) Resulting noise spectrum using interpolation bandwidth $f_n < 2f_{sc}$

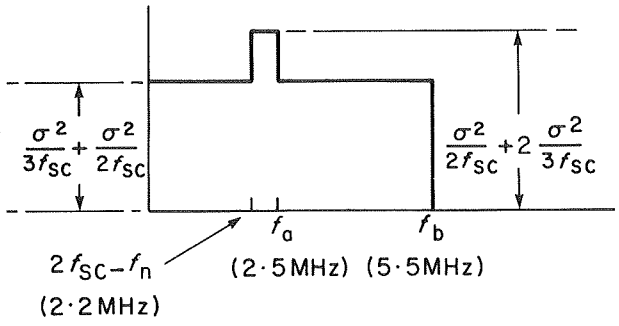


Fig. 6 - 3 f_{sc} to 2 f_{sc} conversion:– noise spectrum with $f_n = 3f_{sc}/2 = 6{\cdot}65$ MHz

5. Sub- to super-Nyquist sampling

This situation is complicated by the fact that the quantising noise embedded in the signal sampled at a sub-Nyquist frequency depends on how it was derived. The 2 f_{sc}-sampled signal may be generated by using analogue comb filtering and sampling; however, it is currently being suggested that a more convenient method is to sample at 4 f_{sc} and down-convert, as described in Section 4. For the purposes of analysis it is assumed that the 2 f_{sc}-sampled signal is derived from a 4 f_{sc} signal; when the 2 f_{sc} signal is in fact directly coded, the situation is equivalent to having an original 4 f_{sc} signal of infinite accuracy from which the 2 f_{sc} signal is derived.

5.1. Twice- to four-times-subcarrier sampling

Quantising-type noise can enter the system at three points:

(i) Sampling the original signal at four times subcarrier frequency.
(ii) Truncating the down-converted (2 f_{sc}) signal after comb filtering.
(iii) Truncating the up-converted (4 f_{sc}) signal prior to the d.a.c.

Except for (iii), this is equivalent to the situation discussed in Section 4.1, namely conversion from four times to twice subcarrier sampling. The additional component of the noise power is therefore that introduced when the 4 f_{sc} signal is truncated; this has a power spectral density of $\sigma_3^2/2\,f_{sc}$ across the band of interest. The total noise power emanating from this process is therefore $\sigma_3^2 f_b/(2f_{sc})$.

5.2. Twice-subcarrier to super-Nyquist

Provided that no significant truncation occurs while converting to the signal sampled at f_3 ($>2f_b$), for example by using 4 f_{sc} as an intermediate sampling frequency, the situation is similar to that discussed above in Section 5.1; however, the final truncation noise power is now $2\sigma_3^2 f_b/f_3$.

5.3. Typical values of increase in quantising noise

Assuming equal quantising accuracies throughout, a signal sampled at 4 f_{sc} derived from a digitally generated 2 f_{sc}-sampled signal (in turn, derived from a 4 f_{sc} signal) suffers an increase in noise over the 2 f_{sc} signal of 1·10 dB if no interpolation is used in the 4 f_{sc} to 2 f_{sc} conversion (i.e. every second sample is taken) and 1·25 dB if an interpolation bandwidth of 6·36 MHz is used. Although the latter figure is higher the absolute noise levels in each case (2 f_{sc} and 4 f_{sc} signals) are less; this is evident from the figures for the increase in noise over a directly coded 4 f_{sc}-sampled signal which are 6·49 dB and 6·02 dB respectively. If the 2 f_{sc} signal is directly coded (or derived from a 4 f_{sc} signal of much greater accuracy) the noise in the final 4 f_{sc} signal is 1·76 dB greater than that in the 2 f_{sc} signal and 4·77 dB greater than that of a directly coded 4 f_{sc} signal.

It is possible to compare the noise levels on an absolute basis by referring to the Appendix, Section 8.2.

In changing the sampling frequency from $2f_{sc}$ to $3f_{sc}$ the noise increase over the $2f_{sc}$-sampled signal is 1·42 dB when the $2f_{sc}$ signal is derived from the $4f_{sc}$ signal without interpolation, 1·60 dB when an interpolation bandwidth of 6·36 MHz is used, and 2·22 dB when the $2f_{sc}$ signal is directly coded or derived from a high accuracy $4f_{sc}$ signal. Relative to the noise in a directly coded $3f_{sc}$ signal, the noise figures are 5·55 dB, 5·12 dB and 3·98 dB respectively; these figures give a truer picture of the actual noise levels in the final $3f_{sc}$ signal. It is assumed that in the final conversion to $3f_{sc}$ (from $2f_{sc}$ or an intermediate $4f_{sc}$) the interpolation bandwidth does not exceed 7·80 MHz.

6. Conclusions and discussion of results

The approach in this report has been to illustrate the various mechanisms by which quantising noise is generated within a system which included a change of sampling frequency rather than to exhaustively list the signal-to-noise ratios of various stages in a chain involving frequency changes. The latter approach would involve a large number of variables such as the quantising step magnitudes, video bandwidth, comb filter response profiles etc.

Each system of digital transmission would require its own analysis in the sense that the effect that each stage has on the form of the quantising noise spectra depends on what precedes that stage; it is unlikely that any general conclusions such as 'changing the sampling frequency from four times to twice colour subcarrier frequency degrades the signal-to-noise ratio by X dB' could be formulated.

Values of quantising-noise power (in dBs) arising from various combinations of sampling and re-sampling are listed in the Appendix, Section 8.2. Equal quantising accuracies are used throughout and the effects of differential pulse code modulation in the link are not considered. Figures for other combinations could be worked out using the methods used in this report.

In the preceding sections the assumptions underlying each of the calculations given must be carefully examined and their implications considered. For example, if a $4f_{sc}$ system is used as an intermediate between a $2f_{sc}$ and (say) a $3f_{sc}$ system, the calculations of Section 5.2 presume that the accuracy of the $2f_{sc}$ system is preserved as far as the output of the $3f_{sc}$ signal. This may not be the case if, for example, it included a piece of equipment dedicated to converting from, say, only an 8-bit $4f_{sc}$ signal to an 8-bit $3f_{sc}$ signal.

Another not so obvious possibility of signal degradation occurs when the new sampling frequency is twice subcarrier frequency; if the frequency is then converted to $4f_{sc}$ for comb-filtering purposes, prior to digital-to-analogue conversion then the accuracy of the d.a.c. should be higher than that which would be used for the $2f_{sc}$ signal for no further appreciable degradation to occur. This case is obviously similar to that outlined in the previous paragraph. There is another side to this particular coin, however. If the last stage in a signal chain is a $4f_{sc}$-sampled signal and if it is not required to transmit or record that signal, then a noise advantage will be gained by using a higher accuracy $4f_{sc}$ signal. For example, if 8-bit coding is used in the transmission stages with $3f_{sc}$ or $2f_{sc}$ sampling, and if conversion to $4f_{sc}$ is undertaken for processing applications, the effect of that stage on the overall quantising-noise power will be negligible if 10 bits are used to describe the signal; naturally, digital-to-analogue converters must contain the same accuracy.

7. References

1. DEVEREUX, V.G. Digital video: sub-Nyquist sampling of PAL colour signals. BBC Research Department Report No. 1975/4.

2. DEVEREUX, V.G. Pulse code modulation of video signal: subjective study of coding parameters. BBC Research Department Report No. 1971/40.

3. CATTERMOLE, K.W. Principles of pulse code modulation. Iliffe, London, 1969.

4. BENNETT, W.R. Spectra of quantised signals. *Bell Syst. Tech. J.*, 1948, **27**, 4, pp. 446 – 472.

5. ROBERTSON, G.H. Computer study of quantiser output spectra. *Bell Syst. Tech. J.*, 1969, **48**, 7, pp. 2391 – 2403.

8. Appendix

8.1. Quantising noise of a sampled signal

In this section the quantising noise spectral density of the sampled signal is derived, and also the total quantising noise power in the Nyquist band. For mathematical purposes in this Appendix the noise will be considered as extending over the frequency range $-\infty$ to $+\infty$.

Assuming the spectral density of the quantising noise prior to sampling is $N_o(f)$ then the total noise power is

$$\int_{-\infty}^{\infty} N_o(f)\,df = \frac{\Delta^2}{12}$$

If this signal is now sampled at a frequency f_s, the sampled quantising noise has a spectral density

$$N(f) = \sum_{-\infty}^{\infty} N_o\,(f - kf_s) \qquad \text{(as shown in Fig. 7).}$$

The total noise power in the Nyquist band is therefore

$$\int_{-f_{s/2}}^{f_{s/2}} N(f)\,df$$

$$= \int_{-f_{s/2}}^{f_{s/2}} \sum_{k=-\infty}^{\infty} N_o(f - kf_s)\,df$$

$$= \sum_{k=-\infty}^{\infty} \int_{-f_{s/2}}^{f_{s/2}} N_o(f - kf_s)\,df$$

Since

$$\int_{-f_{s/2}}^{f_{s/2}} N_o(f - kf_s)\,df$$

$$= \int_{-f_{s/2}+kf_s}^{f_{s/2}+kf_s} N_o(f)\,df$$

then

$$\int_{-f_{s/2}}^{f_{s/2}} N(f)\,df = \sum_{k=-\infty}^{\infty} \int_{-f_{s/2}+kf_s}^{f_{s/2}+kf_s} N_o(f)\,df$$

$$= \int_{-\infty}^{\infty} N_o(f)\,df$$

since all the ranges in the integrals are 'end-to-end'.

Therefore $\int_{-f_{s/2}}^{f_{s/2}} N(f)\,df = \frac{\Delta^2}{12}$ is the total quantising noise power of the sampled signal in the Nyquist band.

For the purposes of describing baseband noise (as in Section 2) negative frequencies are eliminated and $N(f)$ is doubled: thus the total power in the Nyquist band is then

$$\int_{0}^{f_{s/2}} 2N(f)\,df = \frac{\Delta^2}{12}$$

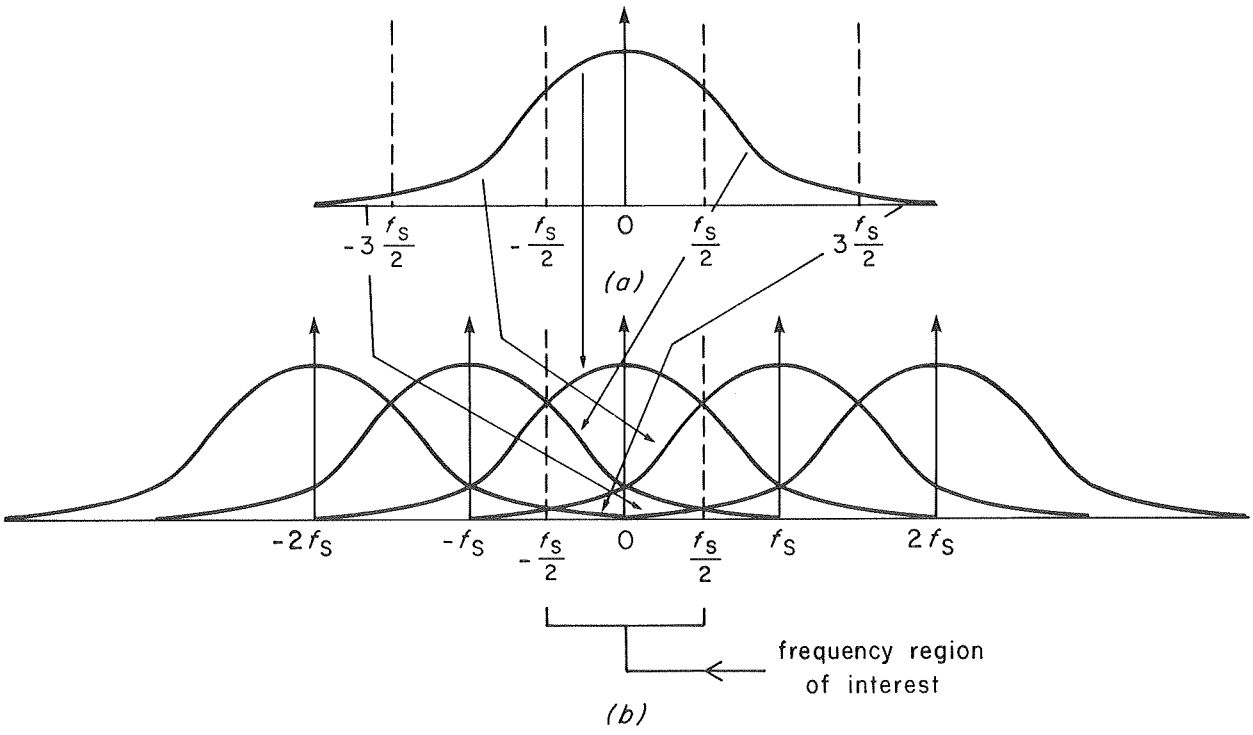


Fig. 7 - Pre- and post-sampling quantising-noise spectra

(*a*) Quantising-noise power-spectrum prior to sampling

(*b*) Noise-power spectrum after sampling. ('Sidebands corresponding to different harmonics of sampling frequency are superimposed.)

8.2. Typical quantising noise levels

The following numbers are convenient 'starting points' to be used in conjunction with the figures given in the examples in the main body of the report.

The r.m.s. quantising noise values in the video band (0 to 5·5 MHz) are listed when 8 bits per sample are used to describe the PAL signal assuming that the available coding range extends from sync. tips to peak subcarrier level. The figures quoted do not tell the whole story: some noise spectra are not flat and hence the net subjective effect may not be directly related to the dB figures. The dB values are relative to the black-level to peak-white signal swing, usually 0·7V.

The quantising noise arising from a single basic coding process is as follows:

$4f_{sc}$ = −56·10 dB

$3f_{sc}$ = −54·85 dB

$2f_{sc}$ = −53·09 dB

The relative figures calculated in the main body of the report for converted signals therefore correspond to the following absolute figures; they apply to 8-bit signals at both the input and the output:

$4f_{sc}$ derived from $3f_{sc}$:	−52·42 dB
$3f_{sc}$ derived from $4f_{sc}$(P):	−52·42 dB
$2f_{sc}$ derived from $4f_{sc}$(N):	−50·72 dB
$2f_{sc}$ derived from $4f_{sc}$(I):	−51·33 dB
$2f_{sc}$ derived from $3f_{sc}$(I):	−50·87 dB
$4f_{sc}$ derived from $2f_{sc}$(D, N):	−49·61 dB
$4f_{sc}$ derived from $2f_{sc}$(D, I):	−50·08 dB
$4f_{sc}$ derived from $2f_{sc}$(A):	−51·33 dB
$3f_{sc}$ derived from $2f_{sc}$(D, N):	−49·30 dB
$3f_{sc}$ derived from $2f_{sc}$(D, I):	−49·73 dB
$3f_{sc}$ derived from $2f_{sc}$(A):	−50·87 dB

(P): Using the preferred interpolator bandwidth, $f_n < 7{\cdot}8$ MHz.

(N): No interpolation used in the $4f_{sc}$ to $2f_{sc}$ conversion.

(I): Using an interpolation bandwidth of 6·36 MHz.

(D): $2f_{sc}$ signal derived from a digital signal at $4f_{sc}$.

(A): From an analogue $2f_{sc}$ signal.

Obviously it is possible to calculate 'triple' or even higher combinations. The list of multiple combinations of direct interest is limited to those starting with $4f_{sc}$ sampling. The use of $4f_{sc} \rightarrow 2f_{sc} \rightarrow 3f_{sc}$ is in fact identical to $2f_{sc}$ (digitally generated) $\rightarrow 3f_{sc}$. Also the case of $4f_{sc} \rightarrow 3f_{sc} \rightarrow 2f_{sc}$ will result in the same total noise power as $3f_{sc} \rightarrow 4f_{sc} \rightarrow 2f_{sc}$ which is in fact $2f_{sc}$ derived from $3f_{sc}$ using $4f_{sc}$ as an intermediate, giving a total noise power of −50·87 dB.

All the figures given above should be increased by 6·02 dB per every bit less used: for example a $3f_{sc}$ signal using 7 bits would have a quantising noise level of −48·83 dB.

Allowances should be made for any reduction in the coding range to allow 'headroom' in the a.d.c.

In conclusion, it is worth remembering that these figures apply to p.c.m. (uniform coding) systems and are not applicable to d.p.c.m. systems, although they may apply to 'quiet' picture areas when d.p.c.m. is used.

Printed by BBC RESEARCH DEPARTMENT, Kingswood Warren, Tadworth, Surrey, KT20 6NP